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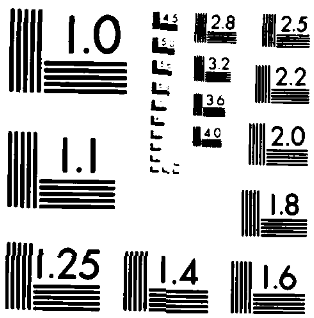

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ANATOMY OF SOME TIME SERIES MODELS

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ABSTRACT

Most naturally occurring data are serial in space or time. With randomized designs, analysis which ignores the serial structure is possible. When the ordering of the data is not at our disposal adequate models must take specific account of serial structure and allow for error dependence, possible non-stationarity, time trend removal, dynamic relationships between variables, feedback between variables, and choice of dependent and independent variables. Stochastic difference equations supply a useful class of serial models. Some aspects of their structure are illustrated with practical examples.

AMS (MOS) Subject Classification: 62M10

Key Words: Time Series Models, Stochastic Difference Equations, Error dependence, non-stationarity, dynamic relationships, time trend removal, feedback, choice of dependent or independent variables.

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ANATOMY OF SOME TIME SERIES MODELS

GEORGE E. P. BOX

INTRODUCTION

Many classical statistical models assume that data $y_1, y_2, \ldots, y_n$ may be directly represented by independently and identically distributed variables. Indeed, some of us have so long been exposed to this idea, in courses, textbooks, and papers that we almost automatically write a relation for the joint distribution of random variables $y_1, y_2, \ldots, y_n$ as

$$p(y_1, y_2, \ldots, y_n) = \prod_{i=1}^{n} f(y_i) \tag{1}$$

and we almost believe it.

In particular, models are frequently employed of the form

$$y_u = f_u + e_u \tag{2}$$

where y_u is a "dependent" (output) variables, f_u is a function of "independent" (input) variables and parameters, and e_u is an error. It is then common to suppose that

$$e_u = a_u \tag{3}$$

where $a_{u-1}, a_u, a_{u+1}, \ldots$ is a sequence of random variables supposed independently and identically distributed about zero. Throughout this paper we shall call such a sequence "white noise".

These assumptions are, of course, sweeping ones. When they are not approximately true, results derived from their use may be very seriously in error. Alternatively, to take a more positive view, the explicit modeling of serial dependence can provide answers to many important practical problems in forecasting, feedback control, the estimation of transfer function relationships, and intervention analysis.

In these days statisticians do not ignore serial dependence, however, there is perhaps a tendency to behave as if our subject could be divided into two pieces - one part concerned with problems readily recognized as time series, where specific allowance is made

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for serial dependence, and the other part, consisting of everything else, where models like (1), (2), and (3) may be safely employed. It seems doubtful, however, whether any such distinction can be made.

UBIQUITY OF SERIAL DEPENDENCE, NON-STATIONARITY, AND NON-HOMOGENEITY

The attractive feature of independence is its mathematical tractability, not its direct association with familiar phenomena within our experience. The data which we mentally process in the conduct of our daily lives is highly serially dependent not independent. To see how important this serial nature of our experience is, imagine viewing the 86,400 separate frames of a one hour movie film after they had been rearranged in random order. It is indeed the serial dependence of our experience which makes it possible to conduct our lives in a rational manner. In particular, it allows us continually to project recent experience and so to make mental forecasts. Comparison of what is expected in the immediate future with actual experience leads to appropriate adjustment of ideas and behavior.

Serial Dependence in Agricultural Field Trials

Although the normal linear model with independently and identically distributed errors is often used as a framework for the analysis of agricultural field trials it is doubtful for example whether R. A. Fisher would place any faith in such an analysis unless the design had been randomized. Only then, he asserted, could such a set-up supply an approximation to the randomization analysis. There are many reasons for randomization but certainly the need to cope with the serial dependence* which would be expected between errors of adjacent plots is an important one.

The principles enunciated in R. A. Fisher's book "The Design of Experiments" are consonant, not with a world of independent errors and homogeneous experimental material, but with inhomogeneity and even non-stationarity. Thus the book is careful to confine

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* Methods for analyzing field trial data which take direct account of spatial correlation have been discussed, for example, by Bartlett (1978).

itself to comparative experiments -- not absolute experiments and the estimates of desired contrasts are made from randomized comparisons within blocks of size eight or less. Study of uniformity data such as that of Wiebe (1935), Figure 1, shows the wisdom of such principles.

<u>Serial Dependence in Industrial Data</u>

Evidence that non-stationary error structures are also to be expected for industrial data is provided, for example, by the devices employed in the process industries for feedback control. Suppose X_t refers to the level, at time t, of an input variable that can be manipulated to compensate for a deviation e_t from some desired output target. A form of feedback regulation commonly found effective employs a control equation having "proportional plus integral" terms. For discrete data it is thus of the form

$$X_t = k_0 e_t + k_1 \sum_{n=0}^{t} e_u \qquad (4)$$

where k_0 and k_1 would normally be of the same sign.

After differencing (4) becomes

$$C(X_t - X_{t-1}) = e_t - \theta e_{t-1} \qquad (5)$$

with $$C = (k_0 + k_1)^{-1} \qquad \theta = Ck_0 \quad .$$

If the control is effective then e_t will be a stationary process implying that X_t follows a non-stationary process. But X_t is the compensation for the disturbance, which consequently must also be non-stationary.

It is further possible to show that, if the dynamic relation between input and output can be approximated by a first order linear difference equation, then (4) can provide minimum mean square error control for a <u>non-stationary</u> disturbance z_t modeled by

$$z_t - z_{t-1} = a_t - \theta a_{t-1} \quad . \qquad (6)$$

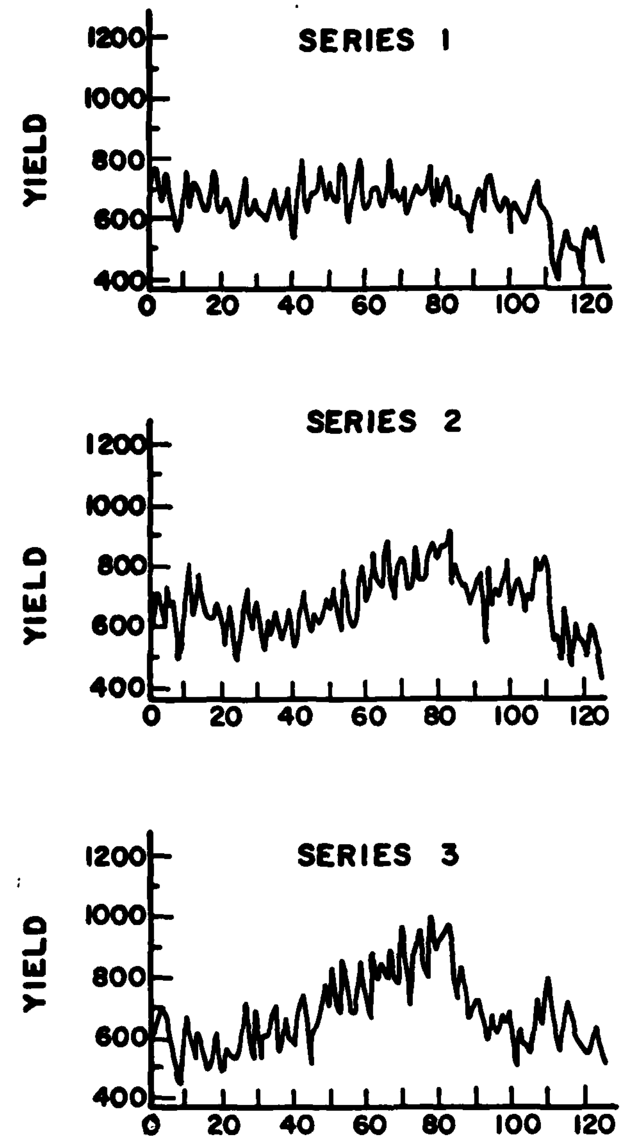

Figure 1: Wheat yields from three sequences of 125 plots (Weibe's data).

MODELING SERIAL PHENOMENA

Randomization is impossible when the ordering of the data in time or space is not at our disposal. It then becomes necessary to model the special characterisitcs of the system arising from its serial nature. These characteristics include

error dependence and possible non-stationarity

existence of time trends

dynamic relationships between variables

feedback

choice of dependent and independent variables.

In the remainder of this paper we discuss the nature of the models employed to accommodate these characteristics. For illustration examples of fitted models are given. However, our purpose here is only to discuss model structure, so details of the model building process are not presented but can be found in accompanying references.

Linear Difference Equations

The crucial first step in showing how linear difference equations might be used in modeling serial data was taken by Yule (1927).

A linear difference equation such as

$$y_t = \delta y_{t-1} + \omega_0 x_{t-1} + \omega_1 x_{t-2} \qquad (7)$$

can, with suitable choices of the coefficients $(\delta, \omega_0, \omega_1)$, represent a dynamic relationship between an input x and an output y. Using B for a backshift operator we can write the relation (7)

$$(1-\delta B)y_t = (\omega_0+\omega_1 B)Bx_t \quad . \qquad (8)$$

More generally with $\delta(B)$ and $\omega(B)$ finite polynomials in B any such linear difference equation may be written

$$\delta(B)y_t = \omega(B)x_t \quad . \qquad (9)$$

For a stable system with the zero's of $\delta(B)$ outside the unit circle we can write (9) as a "distributed lag" model

$$y_t = \frac{\omega(B)}{\delta(B)} x_t = v(B)x_t = \sum_{j=0}^{\infty} v_j B^j x_t = \sum_{j=0}^{\infty} v_j x_{t-j} \tag{10}$$

where $v(B)$ is the transfer function and $\{v_j\}$ is the impulse response function of the system. The impulse response weights v_j thus determine the nature of the linear aggregate $\Sigma v_j x_{t-j}$ which is transferred to the output by the dynamic model.

Stochastic Difference Equations

Stochastic difference equation models suppose that a time series $\{z_t\}$ can be represented as a realization of the output from such a dynamic model in which the input is white noise. Thus

$$\phi(B)z_t = \theta(B)a_t \tag{11}$$

where

$$\phi_1(B) = 1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p, \quad \theta(B) = 1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q \tag{11a}$$

are called autoregressive and moving average operators, respectively.

A model of this kind is often called an autoregressive-moving average model of order (p,q) or an ARMA(p,q) model, see for example Box and Jenkins (1976).

If the zero's of $\theta(B)$ all lie outside the unit circle then we can write the model in invertible form

$$a_t = \frac{\phi(B)}{\theta(B)} z_t = \pi(B)z_t = (1-\pi_1 B-\pi_2 B^2-\ldots)z_t = z_t-\bar{z}_{t-1} \tag{12}$$

where

$$\bar{z}_{t-1} = \sum_{i=1}^{\infty} \pi_i z_{t-1} \quad . \tag{13}$$

Thus, given the model and data up to time $t-1$ the conditional distribution of the next, but not yet available, observation z_t has mean $\bar{z}_{t-1}$ and variance σ_a^2. It is now elementary to calculate also the conditional means and variances for future observations $2, 3,\ldots,\ell$ steps ahead and so provide a basis for forecasting.

As a specific example consider the process (6) which can be written

$$(1-B)z_t = (1-\theta B)a_t \quad . \tag{14}$$

Then

$$a_t = \frac{(1-B)}{1-\theta B} z_t = \{1 - (1-\theta)(1+\theta B+\theta^2 B^2+\ldots)\}z_t = z_t-\bar{z}_{t-1} \tag{15}$$

and

$$\bar{z}_{t-1} = 1-\theta \sum_{i=1}^{\infty} \theta^i z_{t-1} \quad . \tag{16}$$

Thus, for the particular model (14), $\bar{z}_{t-1}$ is an exponentially weighted moving average (ewma) of past data ending at the $t-1^{th}$ observation. The conditional distribution of z_t has this ewma for its mean and has variance σ_a^2.

In the model (14), $\phi(B) = 1-B$ has a zero on the unit circle. By allowing $\phi(B)$ in the general model to have zero's on as well as outside but not inside the unit circle a valuable class of models is obtained for representing non-stationary systems. Also seasonal models of period s can often be represented by using the factorization

$$\phi(B) = \phi_1(B)\phi_s(B^s) \qquad \theta(B) = \theta_1(B)\theta_s(B^s) \quad .$$

Some of the issues are clarified if we consider the problems arising when, as occasionally still happens, ordinary least squares is applied to the analysis of time series data.

For first illustration we reconsider some data plotted in Figure 2 and first studied by Coen, Gomme, and Kendall (1969). These were quarterly data on

z_{1t}: Financial Time Share Index

z_{2t}: U. K. Car Production

z_{3t}: Financial Times Commodity Price Index.

The original authors were interested in predicting z_1 using lagged values of z_2 and z_3 in a linear regression equation which allowed for possible deterministic linear trends in each of the variables.

With e_t a random error, their model can be written

$$y_t = \alpha+\beta_0 t+\beta_1 x_{1t}+\beta_2 x_{2t}+e_t \tag{17}$$

where

$$y_t = z_{1t} \quad , \quad x_{1t} = z_{2,t-6} \quad , \quad x_{2t} = z_{3,t-7} \quad . \tag{17a}$$

Denoting fitted values by $\hat{y}_t$, $\hat{x}_{1t}$, and $\hat{x}_{2t}$, after estimation of a supposed linear time trend in each variable, this model is of the required postulated form

$$y-\hat{y}_t = \beta_1(x_1-\hat{x}_{1t}) + \beta_2(x_2-\hat{x}_{2t}) + e_t \quad . \tag{18}$$

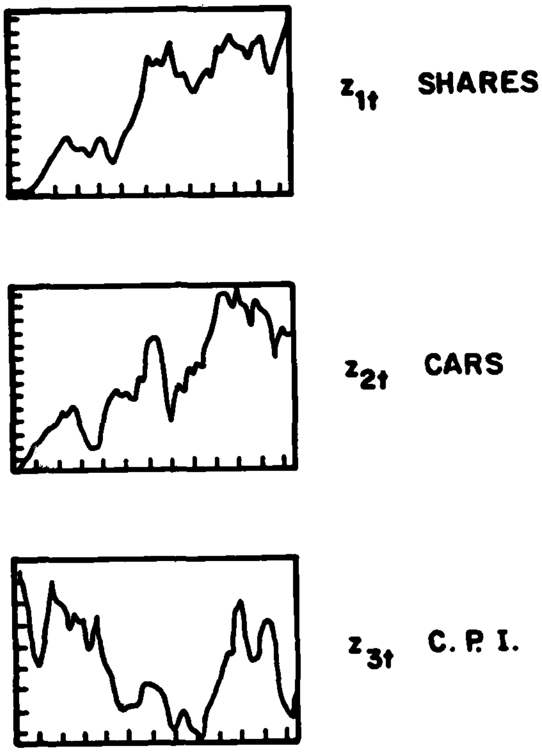

Figure 2: Share Index Data

Column (a) of Table 1 shows the results when equation (17), or equivalently (18), is fitted by ordinary least squares. Both $\hat{\beta}_1$ and $\hat{\beta}_2$ appear enormously significantly different from zero leading to the conclusion that the stock price z_1 can usefully be predicted 6 quarters in advance from knowledge of z_2 and z_3.

Table 1

	(a)	(b)	(c)
Error Model	$e_t = a_t$	$e_t = \phi e_{t-1} + a_t$	$e_t - e_{t-1} = a_t - \theta a_{t-1}$
$\hat{\beta}_1 \times 10^5$	4.7(0.4)	1.8(0.9)	1.6(0.9)
$\hat{\beta}_2 \times 10$	- 6.1(0.5)	- 1.9(1.2)	- 1.2(1.2)
$\hat{\sigma}_a^2$	497	298	321
		$\hat{\phi} = 0.82(0.10)$	$\hat{\theta} = -0.06(0.15)$

Table 1. Estimates of parameters with (standard error) for the model (17)

(a) ordinary least squares

(b) with ϕ in (19) estimated simultaneously from the data

(c) with non-stationary disturbance model (20).

Modeling Serial Dependence of Errors

A numb.. of models which allowed for possible serial dependence of the errors e_t were fitted by Box and Newbold (1970) using maximum likelihood.

In particular the data were consistent with a first order autoregressive model

$$e_t = \phi e_{t-1} + a_t \quad . \tag{19}$$

The profound differences in inference that result when ϕ is not forced to equal zero (ordinary least squares) but is estimated from the data simultaneously with the regression coefficients may be appreciated from comparison of columns (a) and (b) in Table 1 where it will be seen that the previous overwhelming significance of $\hat{\beta}_1$ and $\hat{\beta}_2$ now disappears.

Of the standard assumptions that errors are Normally, Identically and Independently distributed it has been the first two that have received most attention in the literature. It is a measure of the importance of serial dependence that it would require very dramatic departures from normality and/or identicality to influence the estimates and their standard errors in the spectacular manner seen in the comparison of columns (a) and (b) in Table 1.

Non-Stationarity and the Elimination of Time Trends

The autoregressive noise model $e_t = 0.8e_{t-1} + a_t$ is stationary with respect to time, however, $\hat{\phi} = 0.82(0.10)$ is close to unity, for which value the model become non-stationary. Notice also that equation (17) allows for linear dependence of all the series on time with the evident intention of eliminating (non-stationary) time trends.

Now there is no a priori reason for believing in the reality of systematic linear components in the three series. Indeed over longer stretches of time we should expect each series to sometimes trend upwards and sometimes downwards.

A non-stationary disturbance model consistent with the data is in fact of the form of (14)

$$(1-B)e_t = (1-\theta B)a_t \quad . \tag{20}$$

From column (c) of Table 1 we see that the analysis conducted with this noise model gives results similar to those obtained with the autoregressive model. In fact, in this model $\hat{\theta}$ is close to zero. Thus both this non-stationary model and the autoregressive model are pointing to a random walk structure for the error. However, let us continue to consider the more general situation where the noise model is of the form of (20) but θ is not necessarily close to zero.

We are most familiar with the behavior and implications of models for which the errors follow a white noise process and ordinary least squares is appropriate. A useful device in studying models with a more complex noise structure, therefore, is to transform the model to the familiar white noise form.

From (15)

$$a_t = \pi(B)e_t = e_t - \bar{e}_{t-1} \tag{21}$$

where $\bar{e}_{t-1}$ is an exponential moving average of the e's terminating at t-1 with smoothing constant θ.

Operating on both sides of equation (17) with $\pi(B)$ we now obtain

$$y_t - \bar{y}_{t-1} = \beta_0' + \beta_1(x_{1t} - \bar{x}_{1,t-1}) + \beta_2(x_{2t} - \bar{x}_{2,t-1}) + a_t \tag{22}$$

where for example $x_{1t} - \bar{x}_{1,t-1}$ is the deviation of x_{1t} from the exponentially smoothed value $\bar{x}_{1,t-1}$. This equation (22) may now be compared with (18).

We seen then that the fitting of model (17) with non-stationary noise (20) is equivalent to fitting the model (22) by least ordinary squares while allowing for elimination of stochastic trends by exponential smoothing. More generally the model noise structure would indicate precisely <u>what kind</u> of smoothing should be used for elimination of stochastic trend.

<u>Dynamic Relationships Among Variables</u>

Suppose, in the share price data, contributions to z_{1t} from z_{2t} and z_{3t} could be approximated by distributed lag models

$$x_{1t} = v_{12}(b)z_{2t} \qquad x_{2t} = v_{13}(B)z_{3t} \quad .$$

Then the lag structure assumed in (17a) is such that the impulse response functions $\{v_{12,j}\}$ and $\{v_{13,j}\}$ are zero everywhere except at lags $j = 6$ and $j = 7$, respectively, where they take values of unity.

The less restrictive difference equation models of the form of (10) (for example, $v_{12}(B) = \omega_{12}(B)/\delta_{12}(B)$) are clearly more likely to provide adequate dynamic models. Notice that so far as the structure of the resulting models is concerned, after transforming the errors to white noise as before we should have, corresponding to (22):

$$z_1 - \bar{z}_{1,t-1} = \beta_0' + v_{12}(B)(z_{2t} - \bar{z}_{2,t-1}) + v_{13}(B)(z_{3t} - \bar{z}_{3,t-1}) + a_t \quad . \tag{23}$$

The contributions to the response z_1 and z_2 and z_3 would thus be modeled as linear agregates of the deviations from exponentially smoothed values with weights supplied by the impulse response functions of the dynamic systems.

Transfer Function Models

In the particular case of the share index data, analysis fails to show any significant transfer. However, transfer function models of this kind have frequently been effective. For illustration Figure 3 shows hourly data from the Twin Rivers study of house insulation (Pollack and Shoemaker 1978; Socolow 1978). The estimated model*

$$y_t = \frac{0.12+0.42B+0.40B^2}{1-0.33B} x_t + \frac{(1+0.26B)(1-0.86B^{24})}{(1-B)(1-B^{24})} a_t \tag{24}$$

allows for a highly non-stationary seasonal disturbance associated with the daily change in temperature. After this has been taken account of the transfer function relation between outside temperature and attic temperature can be quite accurately estimated.

Particularly in engineering and environmental problems, transfer function models, such as the above, have often proved valuable. Also the transfer function set-up with one or more indicator variables has provided a useful model for intervention studies (see for example Box and Tiao (1975)). For some economic and business data, however, results from transfer function analysis between an output y_t and input $x_{1t}, x_{2t}, \ldots$ has sometimes been disappointing. In particular some selected x_t that seemed a priori to be a promising predictive input has in practice provided only a weak or sometimes a non-detectible relationship. In this connection, however, it must be remembered that denoting the output by y_t an invertible time series model of the form of (11) may be written

$$y_t = \sum_{i=1}^{\infty} \pi_i y_{t-i} + a_t \quad . \tag{25}$$

Thus this univariate time series model can employ the whole past history of y for prediction of future values. Any useful x_t must supply information that is <u>additional</u> to this.

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* Explanation of the model is as follows. The temperature inside the house is going through a large daily cycle which, using a seasonal noise model, could be forecast from the experience of previous days. The outside temperature x_t suitably filtered provides additional current information.

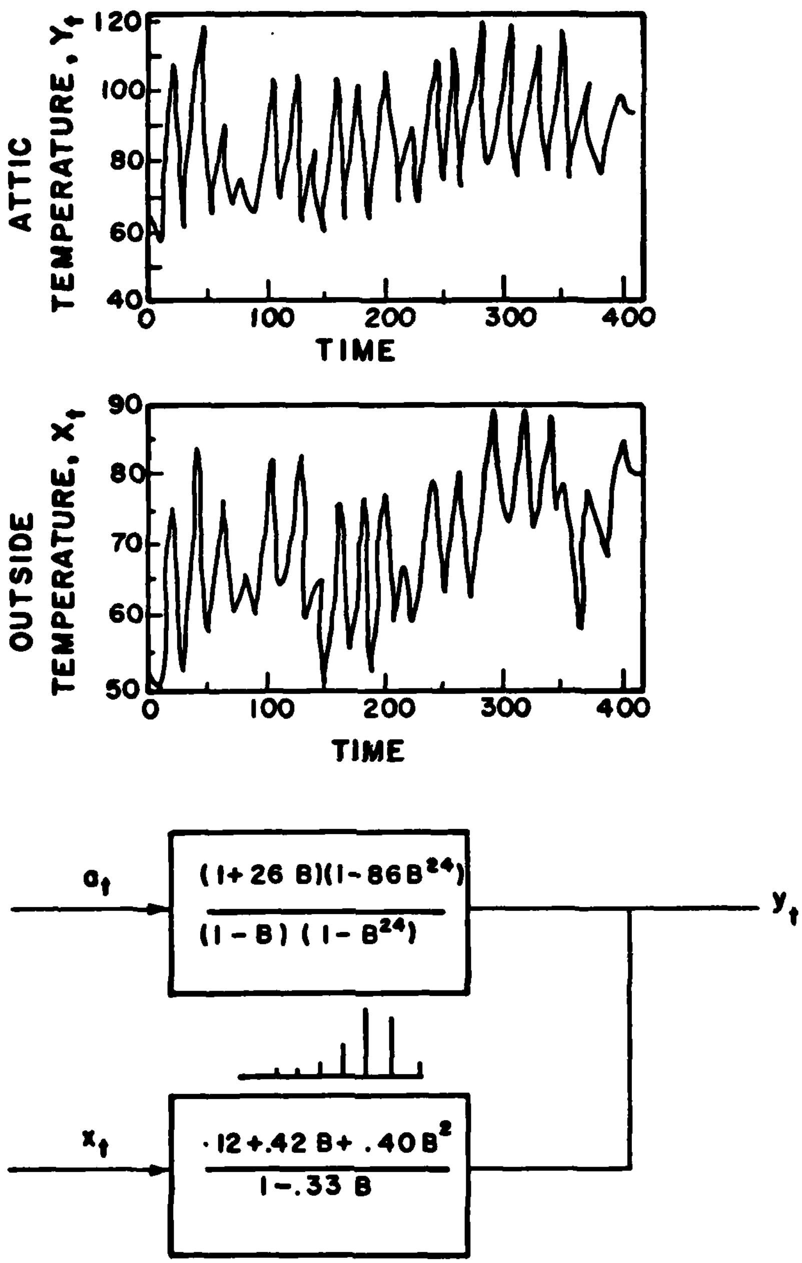

Figure 3: Hourly data outside temperature x_t and attic temperature y_t for an experimental house with estimated model.

Feedback

In any case the usefulness of transfer function models of the kind considered above, is confined to those examples where the relationship is unidirectional. In engineering examples, when feedback occurs, its pattern is usually known from the physical nature of the system and so it can be modeled directly (see for example Box and MacGregor (1976)). However in business and economic systems the nature of the feedback is usually unknown. In the share price example, for instance, the model (17) tacitly assumes the structure indicated in Figure 4(a), when in fact any or all of the relations in Figure 4(b) could occur.

Multiple Time Series Models

Multiple time series models can provide, among other things, a general framework within which all possible feedback relations between k series can occur. They thus present possible means of discovering the feedback from the data. In particular, a class of vector models may be employed which parallels the univariate models of Equation (11) but with $\underset{\sim}{z}_t$ and $\underset{\sim}{a}_t$ now k-dimensional column vectors and $E(\underset{\sim}{a}_t\underset{\sim}{a}_t') = \underset{\sim}{\Sigma}_a$. The autoregressive and moving average parameters are replaced in these models by $k \times k$ matrices.

For example, a first order autoregressive, first order moving average model for $k = 2$ series would be of the form

$$(\underset{\sim}{I}-\underset{\sim}{\phi}B)\underset{\sim}{z}_t = (\underset{\sim}{I}-\underset{\sim}{\theta}B)\underset{\sim}{a}_t \tag{26}$$

or

$$\begin{aligned} z_{1t} &= \phi_{11}z_{1,t-1} + \phi_{12}z_{2,t-1} + a_{1t} + \theta_{11}a_{1,t-1} + \theta_{12}a_{2,t-1} \\ z_{2t} &= \phi_{21}z_{1,t-1} + \phi_{22}z_{2,t-1} + a_{2t} + \theta_{21}a_{1,t-1} + \theta_{22}a_{2,t-1} \end{aligned} \tag{27}$$

$$\Sigma_a = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} . \tag{28}$$

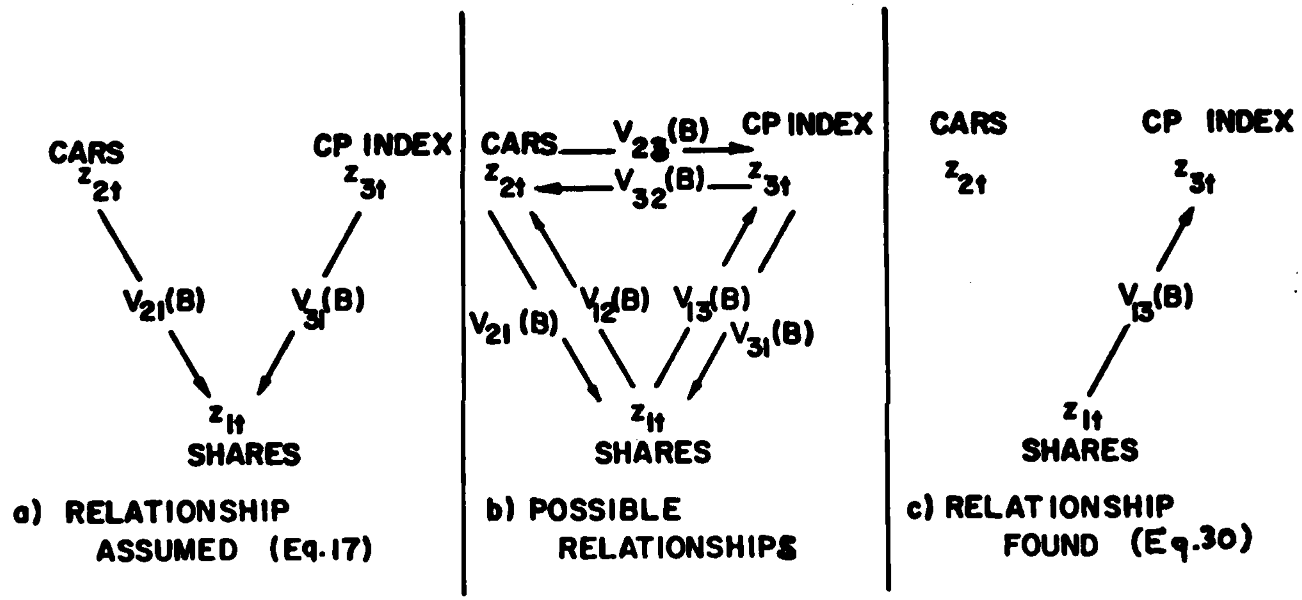

Figure 4: Directional relationships in Share Index Data

In inverted form such a model may be written

$$
\begin{aligned}
z_{1t} &= \sum_{i=1}^{\infty} \pi_i^{(11)} z_{1,t-i} + \sum_{j=1}^{\infty} \pi_j^{(12)} z_{2,t-j} + a_{1t} \\
z_{2t} &= \sum_{i=1}^{\infty} \pi_i^{(21)} z_{1,t-i} + \sum_{j=1}^{\infty} \pi_j^{(22)} z_{2,t-j} + a_{2t} \ .
\end{aligned}
\tag{29}
$$

in which each series can "remember" a linear aggregate of its own past, together with a linear aggregate from the past of the other series.

When a multivariate model of this kind is applied to the share price data (Tiao and Box, (1981)) a trivariate first order autoregressive-first order moving average model is obtained. After appropriate simplication the model implies that the system is approximated by

$$
\begin{aligned}
&(1 - \underset{(.03)}{.98}B)z_{1t} = a_{1t} \\
&(1 - \underset{(.04)k}{.93}B)z_{2t} = \underset{(.1)}{.2} + a_{2t} \\
&(1 - \underset{(.06)}{.83}B)z_{3t} = \underset{(1.1)}{2.8} + \underset{(.23)}{.40}(1 - \underset{(.03)}{.98}B)z_{1(t-1)} + (1 + \underset{(.12)}{.41}B)a_{3t} \ .
\end{aligned}
\tag{30}
$$

$$
\Sigma_a = \begin{bmatrix} .045 & & \\ .024 & .085 & \\ .019 & .023 & .134 \end{bmatrix} \ .
$$

It thus appears that for the share data all three series behave approximately like random walks with slightly correlated innovations. There is (weak) evidence that the share price is a <u>leading</u> indicator at lag 1 for the commodity index. This relation, indicated in Figure 4(c), is in the reverse direction to that assumed in Equation (17).

In general an advantage of this approach is that no prior assumptions need be made about the feedback and dynamic interrelationships between variables and about what should be regarded as an independent variable and what a dependent variable. Rather we can allow the data with rather weak modeling to point to the structure.

Conclusion

The modeling of serial data is important because we live in a serially dependent world.

Considerable success has been achieved in the last 50 years or so in modeling serial dependence using stochastic difference equations.

Much, however, remains to be done. Some important current topics are:

Parsimonious parametrization of multivariate time series models (see, for example, Box and Tiao, 1975; Reinsel, 1983).

Possible value of non-linear models (see, for example, Priestley, 1978).

Treatment of discrepant observations (see, for example, Abraham and Box, 1979).

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Most naturally occurring data are serial in space or time. With randomized designs, analysis which ignores the serial structure is possible. When the ordering of the data is not at our disposal adequate models must take specific account of serial structure and allow for error dependence, possible non-stationarity, time trend removal, dynamic relationships between variables, feedback between variables, and choice of dependent and independent variables. Stochastic difference equations supply a useful class of serial models. Some aspects of their structure are illustrated with practical examples.

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